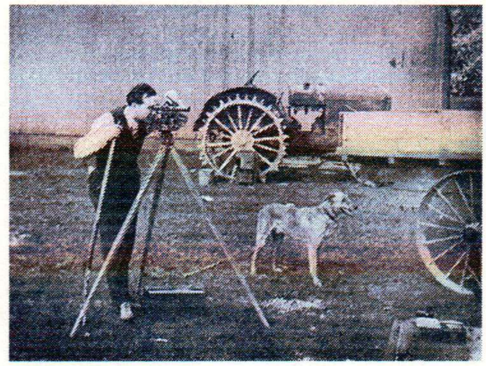
5.8 Triangle Areas by Trig



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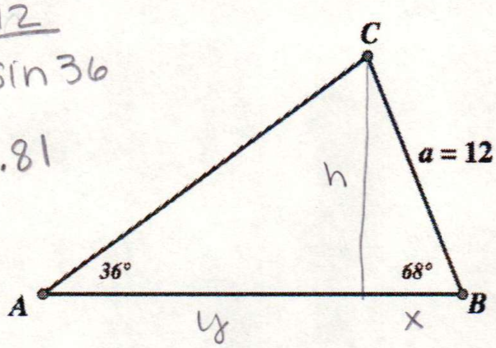
A Practice Understanding Task

Find the area of the following two triangles using the strategies and procedures you have developed in the past few tasks. For example, draw an altitude as an auxiliary line, use right triangle trigonometry, use the Pythagorean theorem, or use the Law of Sines or the Law of Cosines to find needed information.

1. Find the area of this triangle.



$\frac{c}{\sin 76} = \frac{12}{\sin 36}$

$c \approx 19.81$

$\sin 68 = \frac{h}{12}$ $12 \sin 68 \approx 11.13$

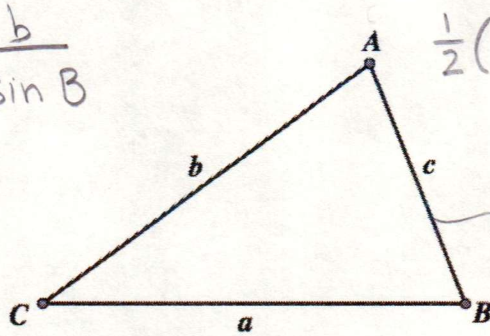
$\cos 68 = \frac{x}{12}$ $12 \cos 68 \approx 4.50$

$\tan 36 = \frac{11.13}{y}$ $\frac{11.13}{\tan 36} \approx 15.32$

$c = 15.32 + 4.50 \approx 19.82$

$\frac{1}{2}(19.82)(11.13) \approx \boxed{110.30}$

2. Find the area of this triangle.



$\frac{a}{\sin A} = \frac{b}{\sin B}$

$a = \frac{b \cdot \sin A}{\sin B}$

$\frac{1}{2}(b \cdot \cos C + c \cdot \cos B)(b \cdot \sin C)$
or
$(c \sin B)$

$\frac{1}{2}\left(\frac{b \cdot \sin A}{\sin B}\right)(c \cdot \sin B)$

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$= \frac{1}{2} b \cdot c \cdot \sin A$

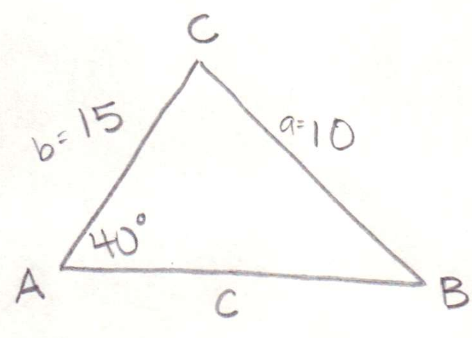
Jumal and Jabari are helping Jumal's father with a construction project. He needs to build a triangular frame as a component of the project, but he has not been given all the information he needs to cut and assemble the pieces of the frame. He is even having a hard time envisioning the shape of the triangle from the information he has been given.

Here is the information about the triangle that Jumal's father has been given.

- Side a = 10.00 meters
- Side b = 15.00 meters
- Angle A = 40.0°

Jumal's father has asked Jumal and Jabari to help him find the measure of the other two angles and the missing side of this triangle.

Carry out each student's strategy as described below. Then draw a diagram showing the shape and dimensions of the triangle that Jumal's father should construct. (Note: To provide as accurate information as possible, Jumal and Jarbari decide to round all calculated sides to the nearest cm—that is, to the nearest hundredth of a meter—and all angle measures to the nearest tenth of a degree.)



Jumal's Approach

- Find the measure of angle B using the Law of Sines
- Find the measure of the third angle C
- Find the measure of side c using the Law of Sines
- Draw the triangle

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Jumal

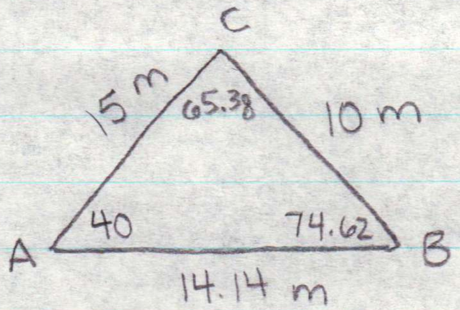
- $\frac{10}{\sin 40} = \frac{15}{\sin B}$ $\quad \sin^{-1}((15\sin 40)/10) \approx 74.62$

$\boxed{\angle B \approx 74.62^\circ}$

- $180 - 40 - 74.62 \approx 65.38$ $\quad \boxed{\angle C \approx 65.38^\circ}$

- $\frac{10}{\sin 40} = \frac{c}{\sin 65.38}$ $\quad \frac{(10 \sin 65.38)}{\sin 40} \approx 14.14$

$\boxed{c \approx 14.14 \text{ meters}}$

- 

Jabari

- $10^2 = 15^2 + c^2 - 2(15)(c)(\cos C)$ ← dont know $\angle C$
should be cos A

$10^2 = 15^2 + c^2 - 2(15)(c)(\cos 40)$
$100 = 225 + c^2 - 30c(.766)$
$0 = 125 + c^2 - 22.98c$
$0 = c^2 - 22.98c + 125$

$$c = \frac{-(-22.98) \pm \sqrt{(-22.98)^2 - 4(1)(125)}}{2(1)} \approx \frac{22.98 \pm \sqrt{28.0804}}{2}$$

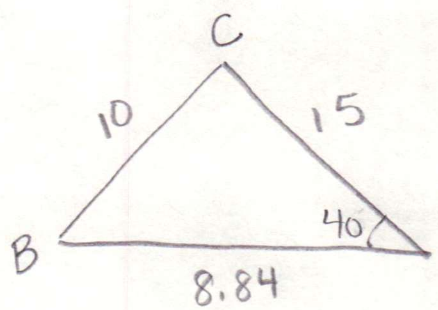
$c \approx 14.14, 8.84$ $\quad c \approx 14.14\text{cm}$ or 8.84 cm

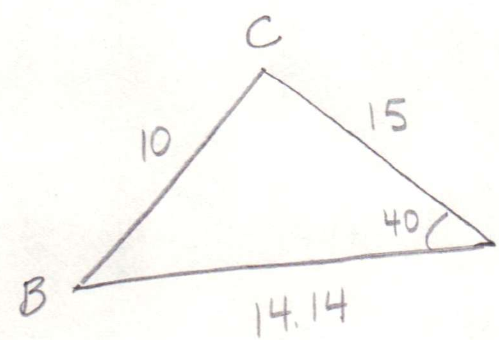
Jabari's Approach

- Solve for c using the Law of Cosines: $a^2 = b^2 + c^2 - 2bc\cos(\overset{A}{\cancel{C}})$

  (Jabari is surprised that this approach leads to a quadratic equation, which he solves with the quadratic formula. He is even more surprised when he finds two reasonable solutions for the length of side c.)

- Draw both possible triangles and find the two missing angles of each using the Law of Sines





$$\frac{\sin 40}{10} = \frac{\sin C}{8.84}$$

$$(8.84 \sin 40) \div 10 = \sin C$$

$$\sin^{-1}(.568) = C$$

$$\angle C \approx 34.61^\circ$$

$$\angle B = 180 - 40 - 34.61$$

$$\angle B \approx 105.39^\circ$$

$$\frac{\sin 40}{10} = \frac{\sin C}{14.14}$$

$$(14.14 \sin 40) \div 10 = \sin C$$

$$\sin^{-1}(.909) = C$$

$$\angle C \approx 65.37^\circ$$

$$\angle B = 180 - 40 - 65.37$$

$$\angle B \approx 74.63^\circ$$

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